

① Use the division algorithm to find $\gcd(104, 524) = d$

Then find c_1, c_2 s.t. $d = 524c_1 + 104c_2$

$$\begin{array}{r} 5 \\ 104 \overline{) 524} \\ \underline{520} \\ 4 \end{array} \quad \rightarrow \quad \begin{array}{r} 26 \\ 4 \overline{) 104} \\ \underline{104} \\ 0 \text{ Stop} \end{array} \quad d = \underline{\underline{4}}$$

$$4 = 524 - (104)(5)$$

$$c_1 = \underline{\underline{1}}$$

$$c_2 = \underline{\underline{5}}$$

② Find $\gcd(9, 128) = d$

Find c_1, c_2 s.t. $d = 128c_1 + 9c_2$

$$\begin{array}{r} 14 \\ 9 \overline{) 128} \\ \underline{126} \\ 2 \end{array} \quad \rightarrow \quad \begin{array}{r} 4 \\ 2 \overline{) 9} \\ \underline{8} \\ 1 \end{array} \quad \rightarrow \quad \begin{array}{r} 2 \\ 1 \overline{) 2} \\ \underline{2} \\ 0 \text{ Stop} \end{array} \quad d = \underline{\underline{1}}$$

$$1 = 9 - (4)(2)$$

↓

$$128 - (14)(9)$$

$$1 = 9 - (4)(128 - (14)(9))$$

$$1 = 9 - (4)(128) + (56)(9)$$

$$1 = (-4)(128) + (57)(9)$$

$$c_1 = \underline{\underline{-4}}$$

$$c_2 = \underline{\underline{57}}$$

1. Use the Euclidean Algorithm to find $d = gcd(104, 524)$
Then find $c_1, c_2 \in \mathbb{Z}$ such that

$$524c_1 + 104c_2 = d$$

Solution: by successive use of the division algorithm we get:

$$\begin{array}{r} 5 \\ 104 \overline{) 524} \\ \underline{520} \\ 4 \end{array} \quad \begin{array}{r} 26 \\ 4 \overline{) 104} \\ \underline{80} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Hence, $gcd(524, 104) = 4$ because 4 is the last nonzero remainder.

Using the next to last division, we can express d as a linear combination of 524 and 104. We find that

$$4 = 524 - 104 * 5$$

Hence, $c_1 = 1, c_2 = -5$

2. Use the Euclidean Algorithm to find $d = gcd(9, 128)$
Then find $c_1, c_2 \in \mathbb{Z}$ such that

$$128c_1 + 9c_2 = d$$

Solution: by successive use of the division algorithm we get:

$$\begin{array}{r} 14 \\ 9 \overline{) 128} \\ \underline{90} \\ 38 \\ \underline{36} \\ 2 \end{array} \quad \begin{array}{r} 4 \\ 2 \overline{) 9} \\ \underline{8} \\ 1 \end{array} \quad \begin{array}{r} 2 \\ 1 \overline{) 2} \\ \underline{2} \\ 0 \end{array}$$

Hence, $gcd(9, 128) = 1$ because 1 is the last nonzero remainder.

Using the next to last division, we can express d as a linear combination of 128 and 9. We find that

$$1 = 9 - 2 * 4$$

The first division tells us that

$$2 = 128 - 9 * 14$$

So $1 = 9 - (128 - 9 * 14) * 4$, which simplifies to

$$1 = 9 * 57 - 128 * 4$$

Hence, $c_1 = -4, c_2 = 57$

1

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